COMPUTER-AIDED DESIGN OF COMMENSURATE AND NON-COMMENSURATE NETWORKS USING STATE-SPACE TECHNIQUES

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Abstract

The state and output equations for networks containing commensurate and non-commensurate delay lines have been derived using topological methods. Computer programs for the analysis and design of these networks have been developed and some of the results are reported. The method is completely general and can be applied to any network topology without any restrictions. Non-commensurate networks offer the possibility of doubling the number of restrictions that can be imposed on the network response. Networks with coupling between the transmission lines can also be solved.

Introduction

A general network containing transmission lines, resistors and voltage and current sources is shown in Figure 1. The sections $S_1 \ldots S_s$ contain the resistor elements and the sources and they are interconnected by the transmission lines. If the delay on each line is the same, the network is commensurate but in general each line will have a different delay. Coupling between all the lines could exist and a common earth plane could exist for some of the lines together with balanced lines in the same network. Two types of degenerate lines exist, these are degenerate voltage (current) lines which are terminated in an ideal voltage (current) source and degenerate short (open) circuited lines which are terminated in a short (open) circuit.

Theoretical Analysis

The state equation of networks containing commensurate delay lines has been derived using topological methods[1]. The equation is of the form

$$b(t+T) = A\, b(t) + B_1\, u_R(t) + B_2\, u_T(t) \qquad (1)$$

where the state vector $b(t)$ is the reflected parameters at all the transmission line ports, $u_R(t)$ and $u_T(t)$ are vectors related to the input voltage and current sources, A is the state matrix and B_1 and B_2 are matrices relating the inputs to the state vector. The order of the state vector $b(t)$ is twice the number n of all the transmission lines.

When the z-transform is applied to (1), the state equation in the z-domain is obtained

$$b(z) = [z\, U_{2n} - A]^{-1} [B_1\, u_R(z) + B_2\, u_T(z)] \qquad (2)$$

where $z = e^{sT} = e^{(\sigma + j\omega)T} = \Sigma + j\Omega$,

T is the commensurate delay of the transmission lines and U_{2n} is a unit matrix of order 2n.

Similarly, the output equation of the form

$$y(t) = C\, b(t) + D_1\, u_R(t) + D_2\, u_T(t) \qquad (3)$$

has been obtained for all possible output vectors $y(t)$.

When the z-transform is applied to (3) the result can be solved with (2) to obtain the transfer function

$$y(z) = [C[z\, U_{2n} - A]^{-1} B_1 + D_1][u_R(z)] + [C[z\, U_{2n} - A]^{-1} B_2 + D_2][u_T(z)] \qquad (4)$$

The topological forms of the matrices A, B_1, C, D_1 and D_2 are valid for commensurate and non-commensurate networks. In the non-commensurate case the delays on the lines could be all difference and the time advanced state vector in (1) should be replaced by $[b_1(t + \alpha_1 T)\; b_2(t + \alpha_2 T) \ldots b_{2n}(t + \alpha_{2n} T)]^T$ where $\alpha_1 \ldots \alpha_{2n}$ are the ratios between the delays on each line and the normalised or "standard" delay T. The normalisation could also be made with respect to any one of the lines in the network.

To obtain the transfer function for non-commensurate networks the matrix $[z\, U_{2n} - A]$ in (4) is replaced by $[\mathrm{diag}(z^{\alpha 1}, z^{\alpha 2} \ldots z^{\alpha 2n}) - A]$

When deriving the state and output equations it is necessary to obtain the scattering matrix of the whole transmission line system normalised to the characteristic resistance matrix R_o. When coupling between the lines does not exist, the matrix R_o is a diagonal matrix of all the self-characteristic resistances of the lines. When coupling exists the off-diagonal elements of R_o are no longer zero. This is the only modification required to include coupling between the lines. In the most general case, coupling would exist between all the transmission lines and all the elements of R_o are non-zero.

Properties of Transmission Line Networks

Given any commensurate network, each line could be divided into an integral number of equal lines and a new commensurate delay is obtained. For example, if each line is divided into three equal lines the state equation can still be written with respect to the new commensurate delay which is one third of the original delay. The fundamental delay T_F is defined as the longest delay for which a state equation exists. The order n of the network is the rank of the state matrix A when the state equation is written with respect to the fundamental delay. The degree of freedom is defined as the number of restrictions that can be imposed on the network response, given the freedom of adjusting the unknown element values.

In a non-commensurate network, it is always possible to choose the delay T such that the ratio between the highest and the lowest α is not greater than 2. In this case, the network can be considered as a perturbation of a prototype commensurate network obtained by setting all the α ratios to 1. The order of the non-commensurate network will be defined as the order of the prototype network and all the delays are written in terms of the commensurate delay. Since the number of unknowns are now the line lengths as well as their characteristic resistances then d = 2n.

In normal networks the output y(t) vector is related to the input vector u(t) by

$$y(t) = \sum_{n=0}^{\infty} a_n u(t-2nT)$$

The fundamental delay T_F in this case is 2T.

Table I gives a comparison between various types of networks, in each case the total number of transmission lines is n.

	n	d
Normal commensurate	n	n
Networks designed by Kuroda identities with r cascaded and s shunt lines	s	s
Non-normal commensurate	2n	n
Normal non-commensurate	n	2n
Non-normal non-commensurate	2n	2n

Table I : The order and the degree of freedom of various types of networks containing n transmission lines.

From Table I it is seen that the order of the response and the degree of freedom are not always the same. Furthermore, in the case of non-commensurate networks, the degree of freedom is twice that of a commensurate network with the same topology.

The Input Data

To formulate the data required by the computer programs an unconnected graph consisting of all the transmission line and resistive edges is drawn for all the sections. A tree is chosen for each section such that it contains the minimum number of transmission line edges, all the degenerate open circuited and current transmission line edges, and none of the degenerate short circuited and voltage transmission line edges. All the unconnected graphs are treated as one graph with a forest consisting of all the trees and a coforest consisting of all the cotrees. In numbering the edges of the graph we start by the transmission line edges in the forest followed by the resistive edges in the forest, the transmission line edges in the coforest and the resistive edges in the coforest. The required input data is listed below

a. The numbers l and m of the transmission line edges in the forest and coforest respectively, and the numbers a and b of the resistive edges in the forest and coforest respectively.

b. *The dynamical transformations D.* This is the matrix describing the interconnections between the elements. The rows of D are assigned to the branches (edges in the forest) and the columns are assigned to the chords (edges in the coforest). Each column in D is a tie-set consisting of the assigned chord and as many branches as necessary.

c. *The row operations matrix K.* This matrix rearranges the rows of the state vector such that the two rows belonging to the same transmission line are interchanged. The matrix K is obtained by assigning its columns to the transmission line edges in the order they appear in the state vector and its rows to the same edges with the two edges belonging to the same line interchanged. An entry of 1 is made at the intersection of the column and row assigned to the same edge and an entry of 0 is made otherwise.

d. *The resistive elements matrix R.* This is a diagonal matrix of all the values of the resistive elements in the order assigned to their edges.

e. *The characteristic resistance matrix R_o.* This is a matrix of the self and mutual characteristic resistances of the system of transmission lines. In the synthesis program the elements of R_o are the initial guesses for the error minimisation procedure.

f. *The delay ratio vector α.* This is a vector of the ratios of the delays on the lines to a standard delay. In the synthesis program the value of these ratios are the initial guesses for the error minimisation procedure.

g. *The input vectors.* These are current and voltage vectors of the sources in each edge.

h. *The output directive.* This directive indicates which output is required.

i. *The required output restrictions.* For the synthesis program the required restrictions on the network output are part of the input data. The maximum number of such restrictions is equal to the degree of freedom, d.

The Analysis and Synthesis Programs

The analysis program uses the input data to construct the state and output equations and calculate the amplitude and phase of the transfer function as a function of ω. The program also calculates the natural frequencies and the zeros of transmission in the z, s and λ(λ = tanh sT) domains.

The synthesis program calculates the required values of the characteristic resistances and the delay ratios to satisfy the constraints on the transfer function. In the case of commensurate networks, the most convenient method of specifying the constraints on the transfer function is to specify the locations of the natural frequencies. In some cases, especially for non-commensurate networks, this may not be possible and the restrictions are applied directly on either the amplitude or phase of the transfer function.

Examples

Some examples are given below. The results of the synthesis program correspond to the norm of the error vector being less than 10^{-8}.

Example I: This is a synthesis problem for the fifth order commensurate network shown in Figure 2. A non-redundant equiripple response is required with 0.5 dB ripple and a cut-off angle ωT of 1. First the required values of the natural frequencies in the z-plane were obtained given that the network has three transmission zeros at +j and -j and two transmission zeros at 0 and at ∞. The characteristic resistances of the lines were calculated and they are given in Figure 2 together with the network response.

Example II: In this example a non-commensurate third order network was designed to meet the following six conditions: (a) A zero of transmission at ωT = 1.5 (b) A zero of transmission at ωT = 1.6 (c) A minimum insertion loss of 50 dB in the range of ωT = 1.5 and 1.6 (d) & (e) Both maxima in the pass-band should be 0.5 dB (2 conditions) (f) The lower cut-off angle ωT is 2.2.

The results are shown in Figure 3 and the specified restrictions are marked on the response curve.

Example III: This example is based on the sixth order inter-digital filter given by Wenzel[2]. The effect of coupling between non-adjacent elements was studied. Coupling between elements 2-4, and 3-5 was taken as 5% of the coupling between adjacent elements. These values were added to Wenzel's network and the results are shown in Figure 4. It is clear that a small amount of coupling between non-adjacent elements has a considerable effect on the network response.

Conclusions

A powerful and accurate method for the analysis and synthesis of transmission line networks has been developed. The method can be applied to commensurate, non-commensurate and coupled networks with no restrictions on the topology. The method can be used in either the time or frequency domains and can be extended to solve networks containing both lumped and distributed elements.

References

1. M. I. Sobhy, "Topological Derivation of the State Equation of Networks Containing Commensurate Transmission Lines," Proc. IEE, vol. 122, no. 12, pp. 1367-1371, 1975.

2. R. J. Wenzel, "Exact Theory of Inter-digital Band Pass Filters and Related Coupled Structures," IEEE Trans. on Microwave Theory and Techniques, vol. MTT-13, no. 5, pp. 559-575, 1965.

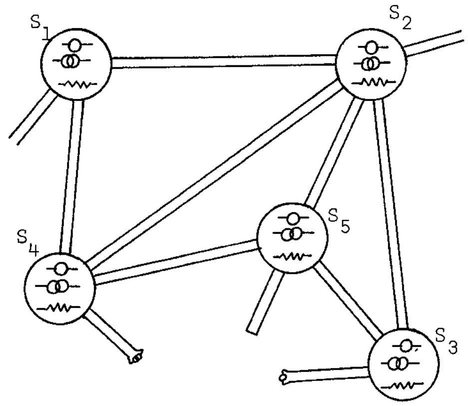

Fig. 1-General transmission line network.

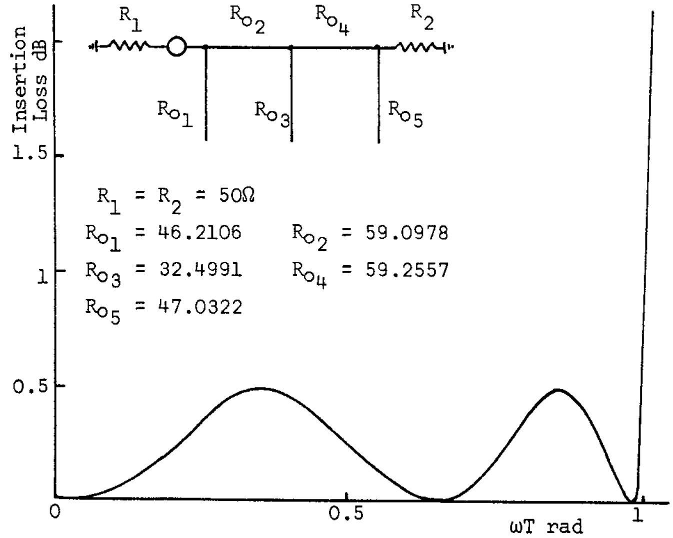

Fig. 2-Non-redundant fifth order filter (Example I).

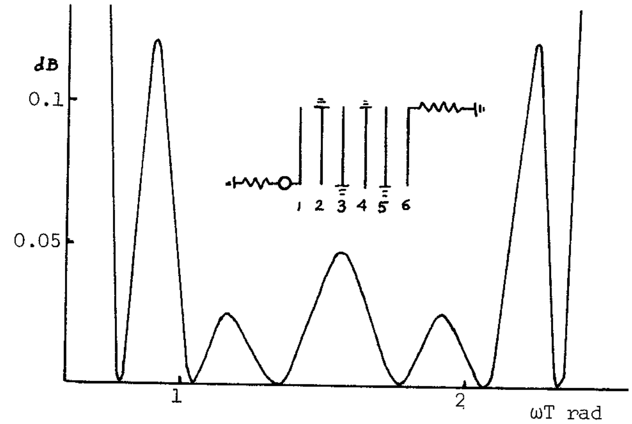

Fig. 4-Effect of coupling between non-adjacent elements (Example III).

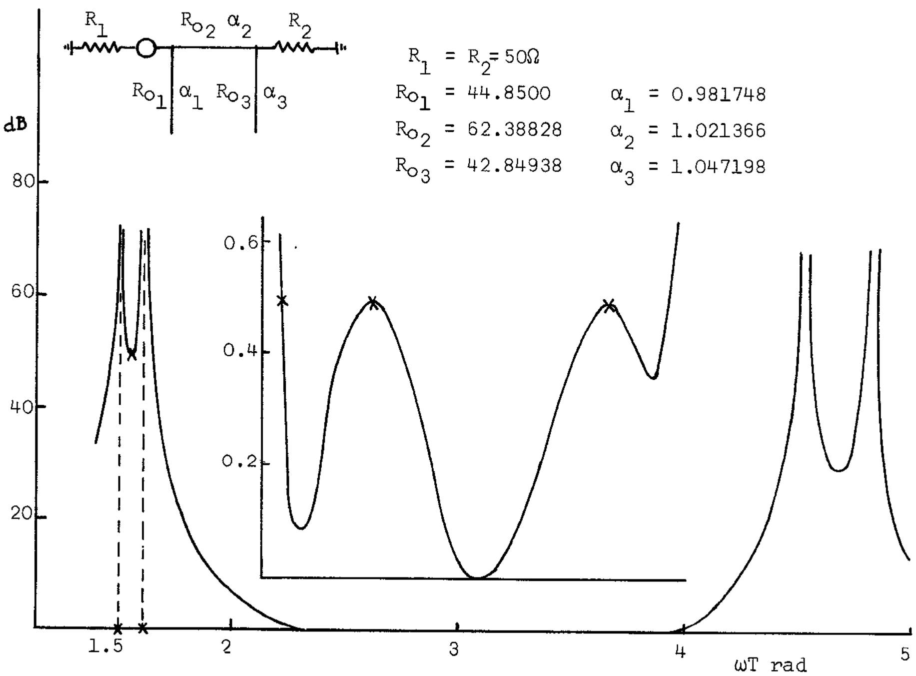

Fig. 3-The realisation of six restrictions on the response of a third order non-commensurate network (Example II).